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Vibration Analysis of a Simply Supported Beam with Multiple Breathing Cracks

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Abstract

The dynamic characteristics of a beam with multiple breathing cracks are studied in this paper. A systematic approach has been adopted in the present investigation to develop theoretical expressions for evaluation of natural frequencies and mode shapes. A simple elastic simply supported beam with two breathing cracks is considered for the dynamic analysis. The stiffness of the cracked beam is found out by using influence coefficients. The influence coefficients are calculated by using castigiliano's theorem and strain energy release rate (SERR).The equation of motion of the beam was derived by using Hamilton's principle. The stiffness and natural frequencies for the multiple cracked beam calculated using eigen value approach. It is seen that due to presence of cracks, the stiffness and natural frequency changes. The mode shapes and the FRF for the uncracked, cracked and breathing cracked cantilever beam also obtained and compared. The mode shape changes considerably due to the presence of crack.

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*Keywords:*Breathing crack, Mode shape, Frequency response (FRF), FEM, Vibration analysis.

Introduction

Beams are one of the most commonly used structural elements in numerous engineering applications (ex. aerodynamic structures, tankers and rotors, etc.,) and experience a wide variety of static and dynamic loads. The crack modelling is an important step in studying the behaviour of damaged structures. The majority of published studies assume that the crack in a structural member always remains open during vibration. The cracks are always open in vibration is not realistic because due to repetitive loads the crack may open or close. The relation between the applied load and the strain energy concentration around the tip of crack is explained by Irwin (1957). Qian et al. (1990) developed a finite element model of an edge cracked beam. In his paper, an element stiffness matrix of a beam with crack is first derived from an integration of stress intensity factors. The finite element model is applied to a cantilever beam with an edge crack, and the eigen frequencies are determined for different crack lengths and

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locations. Douka et al. (2005) investigated time–frequency analysis of the free vibration response of a beam with a breathing crack using the empirical mode decomposition and Hilbert transform for single degree of freedom system (SDOF) of a beam. The influence of a crack in a welded joint on the dynamic behaviour of a structural member is discussed by Chondros and Dimaragonas (1980). The vibration analysis of a cracked beam and cracked shaft have been studied by Dimarogonas et.al (1983).They derived the flexibility matrix of the cross section containing the crack.The vibrational response of the harmonic force of a cantilever beam with cracks of different size and locations has been analysed by Pugno et al. (2000) using harmonic balance method. They analysed and compared the results for the free end dynamic response of beam with numerical integration. The effects of closure of cracks on the dynamics of a cracked cantilever beam have been analysed by Kisa et al. (2000) using the finite element method, component mode synthesis and time stepping scheme. Modelling of a beam with a breathing edge crack subjected to harmonic loading has been investigated by Nandi et al. (2002). They considered the frictionless contact model with single degree of freedom. Andreaus et al. (2007) studied the nonlinear dynamics of a cracked cantilever beam under harmonic excitation using two dimensional finite elements. They considered the frictionless contact model and single degree of freedom to solve the problem and outlined the response with respect to the linear one. Dynamic characteristics of a cantilever beam with transverse cracks are analytically investigated by Behera et al. (2006) using the contour plots for finding out the various effect of cracks. In this thesis finite element method is proposed to model the beams with a breathing crack using fracture mechanics methods. In modelling, the effect of the crack on the deformation of a beam has often been considered similar to that of an elastic hinge (or) plastic hinge. The behaviour of beam with crack and without crack is studied. An investigation of crack depth and its location are provided here directly by using the mode shape analysis.

2. Finite Element Formulation

2.1. Derivation of Shape Function

For finite element formulation, the beam element of fig. (2) is taken here with the length of l and axial load co-ordinate x and transverse local co-ordinate y. The local transverse nodal displacements are assumed as u_{1y} and u_{2y}. The rotations are given by ϕ_1 and ϕ_2. The local nodal forces are given by F_{1y} and F_{2y}. The bending moments are given by m_1 and m_2. For simplicity, the beam is divided into 21 numbers of discretized finite elements as shown in fig. (1).

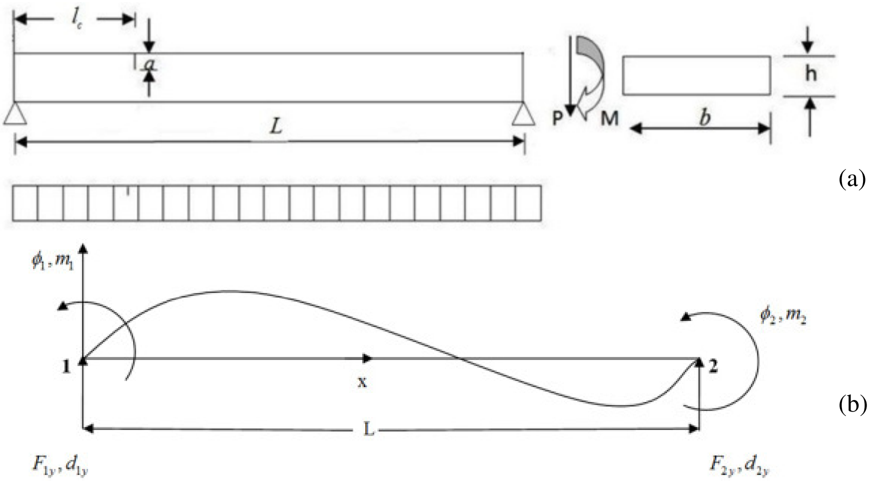

Figure1a. Schematic diagram of a beam. Figure 1b.Beam element with positive

nodal displacements, rotations, forces, and moments.

From fig. 1(b), the transverse displacement variation through the element length to be approximated as a polynomial function of quadratic equation as,

$$v(x) = a_1x^3 + a_2x^2 + a_3x + a_4. \quad (1)$$

The above equation can be rewritten in matrix form of $v(x) = [N]\{\delta\}^e$ as,

$$v(x) = [N_1 \quad N_2 \quad N_3 \quad N_4]\begin{Bmatrix} u_{1y} \\ \phi_1 \\ u_{2y} \\ \phi_2 \end{Bmatrix}. \quad (2)$$

Where,

N_1, N_2, N_3 and N_4 are shape functions for beam element.

$$N_1 = \frac{1}{l^3}\left(2x^3 - 3x^2l + l^3\right),\ N_2 = \frac{1}{l^3}\left(x^3l - 2x^2l^2 + xl^3\right),\ N_3 = \frac{1}{l^3}\left(-2x^3 + 3x^2l\right),\ N_4 = N_2 = \frac{1}{l^3}\left(x^3l - x^2l^2\right).$$

2.2. The Stiffness Matrix for Beam Element

The element stiffness matrix of the beam element can be derived as,

$$[K_e] = \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}. (3)$$

2.3. Mass Matrix for the Beam Element

The shape function is used as the weight functions in Galerkin's method. Substituting the shape functions in the weak form equation which is given as below,

$$\left[W(x)EI\frac{d^3v}{dx^3}\right]_0^l - \left[\frac{dW}{dx}EI\frac{d^2v}{dx^2}\right]_0^l + \int_0^l EI\frac{d^2v}{dx^2}\frac{d^2W}{dx^2}dx - \int_0^l \rho A\omega^2 W(x)v(x)dx = 0, (4)$$

and evaluating the integrals with respect to each of the weighing functions one can obtain the governing equations at a time and rewrite those terms in matrix, one can obtain the element mass matrix of an Euler Bernoulli beam as,

$$[M_e] = \frac{\rho Al}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}. \quad (5)$$

Where, l=Length of the beam element in m, ρ =Density of the beam material in kg/m^3 and A =Area of cross section in m^2. Only a transverse crack under bending has been used to establish the element stiffness matrix.

3. Modelling of Cracks

A cantilever beam of length L, of uniform rectangular cross section $b \times h$ with a crack located at positions l_1 is considered (Fig. 1). The crack is assumed to be open and close periodically. The depth of crack is a. The beam is subjected to transverse & bending load, P & M respectively.

3.1. Local Flexibility of a Cracked Beam under Bending

The presence of a transverse crack of depth a introduces a local flexibility matrix. The dimension of the local flexibility matrix is (2×2), as each side has two degrees of freedom. The off-diagonal elements of the flexibility matrix are considered as coupling elements. According to the principle of Saint Venant, the stress field only affects the region which is adjacent to the crack. The flexibility co-efficient is expressed by means of stress intensity factor which can be easily derived by using Castigiliano's theorem in the linear elastic range. With shearing action neglected, the strain energy of an element without a crack can be written as,

$$W^{(0)} = \int_0^l \frac{M'^2}{2EI} dx. \quad (6)$$

Where,

I =Moment of inertia of the uniform beam, E, E' =Elastic modulus and M =Bending moment due to the applied force.

Total bending moment due to the transverse force can be written as (see fig. (1))

M'=Px+M. (7)

Where

P =Transverse Load and M' =Total Bending moment.

Therefore eq. (6) can be rewritten as,

$$W^{(0)} = \int_0^l \frac{(\mathrm{P}x+\mathrm{M})^2}{2EI} dx. \quad (8)$$

After simplification the strain energy of an element without a crack can be obtained as,

$$W^{(0)} = (M^2 l + MPl^2 + P^2 l^3/3)/2EI. \quad (9)$$

Where, l is the Length of the beam element. The additional strain energy due to the crack can be written as (Tada H (1973), Qian et al. (1990)),

$$W^{(1)} = b\int_0^a [(K_I^2 + K_{II}^2)/E' + (1+\nu)K_{III}^2/E] da. \quad (10)$$

Where $E' = E/(1-\upsilon^2)$ for plain strain and $E' = E$ for plane stress problem condition. E is the Young's modulus of elasticity and ν is the Poisson's ratio. K_I, K_{II}, K_{III} are stress intensity factors for opening, sliding and tearing mode cracks. The amplitude of vibration is assumed to be well below the crack opening due to preloading.

With the action of axial force neglected, the above eq. (10) becomes,

$$W^{(1)} = b\int_0^a \{[(K_{IM} + K_{IP})^2 + K_{IIP}^2]/E'\} da. (11)$$

The stress intensity factors from elementary fracture mechanics are given as,

$$K_{IM} = (6M/bh^2)\sqrt{\pi a}\,F_I(s),\ K_{IP} = (3Pl/bh^2)\sqrt{\pi a}\,F_I(s) \text{ and } K_{IIP} = (P/bh)\sqrt{\pi a}\,F_{II}(s). \quad (12)$$

Where b and h are the cross-section dimensions and a is the crack depth as shown in fig. (1).The relative crack depth, the crack location from fixed end and the relative crack location are denoted as $s = (a/h)$, l_1 and $l' = (l_1/L)$ respectively. The functions $F_I(s)$ and $F_{II}(s)$ are dependent on the crack depth a and are approximated (Stephen H C 1978) by

$$F_I(s) = \sqrt{(2/\pi s)tg\,(\pi s/2)}\,\frac{0.923 + 0.199[1-\sin(\pi s/2)]^4}{\cos(\pi s/2)} \;\&\; F_{II}(s) = (3s-2s^2)\frac{1.122 - 0.561s + 0.08s^2 + 0.18s^3}{\sqrt{1-s}}.$$

Here we consider the $F_I(s)$ for calculating the stress intensity factors for open mode crack. In addition, the crack produces a local additional displacement u_i between the right and left sections of the crack, in a similar way as an equivalent spring. These displacements U_i in the i direction, under the action of force P_i are given, according to Castigiliano's theorem.The flexibility co efficient for an element without a crack is,

$c_{ij}^{(0)} = \partial^2 W^{(0)} \big/ \partial P_i \partial P_j$. (13) Substituting $P_1 = P$; $P_2 = M$; $i, j = 1,2$ in eq. (13) one can obtain the resulting equation as,

$$c_{11}^{(0)} = \frac{l^3}{3EI},\ c_{12}^{(0)} = c_{21}^{(0)} = \frac{l^2}{2EI},\ \left(\because \frac{\partial^2 W^{(0)}}{\partial P \partial M} = \frac{\partial^2 W^{(0)}}{\partial M \partial P}\right) \text{ and } \quad c_{22}^{(0)} = \frac{l}{EI}.$$

Similarly the additional flexibility co efficient for the cracked element can be obtained as,

$c_{ij}^{(1)} = \partial^2 W^{(1)} \big/ \partial P_i \partial P_j$. (14)

Therefore the total flexibility co efficient for the cracked beam is,

$C = c_{ij}^{(0)} + c_{ij}^{(1)}$. (15)

The order of the flexibility matrix is 2×2. From the equilibrium condition of the cracked beam element as shown in fig. 1(b) one can have $(P_i \ \ M_i \ \ P_{i+1} \ \ M_{i+1}) = [T](P_{i+1} \ \ M_{i+1})$. (16)

Where,

[T]=Transformation matrix and is given by

$$[T] = \begin{bmatrix} -1 & -l & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}. \ (17)$$

Using the principle of virtual work, the stiffness matrix of the cracked element can be obtained as,

$[K_c] = [T]^T C^{-1} [T]$. (18)

3.2. Mathematical Modelling of Breathing Crack

The stiffness of a structure with breathing crack changes with respect to the time and it can be expressed as a continuous function of time in the form,

$K(t) = K_c + K_{\Delta c}(1 + \cos \omega_b t)$.

4. Equation of Motion of an Euler Bernoulli Beam

Hamilton's equation can be written as below,

$$\int_{t_1}^{t_2} \left(\delta (T - U) + \delta W_{nc} \right) \ (19)$$

Substituting the values of T ,U and $\delta W_{nc} = f \delta u - c \dot{u} \delta u$ as in eq. (19) one can obtain the equation of motion of a beam as

$$M\ddot{u} + \left(K_c + K_{\Delta c}(1 + \cos \omega_b t)\right) u = 0. \qquad (20)$$

The non-conservative forces are not considered here.

4.2. Modal Analysis

The global stiffness matrices $[K]$ and the global mass matrices $[M]$ for the beam is obtained by means of applying the boundary conditions for the beam as, $U = 0$ and $\theta \neq 0$ for simply supported beam. The natural frequencies (eigen value) and the mode shapes (eigen vector) are obtained by means of using Eigen value analysis using matlab (using the command eig (K, M)) using finite element methodology. The crack only affects the stiffness of the structure. There is a minor difference ($K_{\Delta c}$) between the general stiffness matrixes of the cracked and uncracked beam.

5. Numerical Analysis

For numerical simulations, mild steel beam of total length 0.8m and rectangular cross section 0.05×0.006m is considered. A crack of varying depth is introduced at l_c =0.05, 0.1, 0.2, 0.3, 0.4 and 0.60 from the fixed end. A young's modulus of 2.1×10^{11} N/m^2, a density of 7850 kg/m^3, a Poisson ratio of 0.3 and a damping factor of 0.01 were used. The dynamic response of the beam was obtained. The initial conditions were taken as $u'(0) = 0$ m/s and $u(0) = 0$ m. The numerical results are presented in the tabular & graphical form.

Table 1. Natural frequencies of an open cracked and breathing cracked beam.

Sl no	First natural frequency ω_1 (rad/sec)	Second natural frequency ω_2 (rad/sec)	Third natural frequency ω_3 (rad/sec)
1	138.1514	552.6086	1243.397

Table 2. Natural frequencies of an uncracked beam.

Sl no	Crack location		Crack depth		First natural frequency ω_1 (rad/sec)		Second natural frequency ω_2 (rad/sec)		Third natural frequency ω_3 (rad/sec)	
	l_1	l_2	a_1	a_2	Open crack	Breathing crack	Open crack	Breathing crack	Open crack	Breathing crack
1	0.076	0.114	0.001	0.001	138.149	138.1502	552.5748	552.5917	1243.258	1243.328
				0.002	138.1468	138.1491	552.5443	552.5765	1243.139	1243.268
				0.003	138.1296	138.1407	552.3075	552.4597	1242.214	1242.812
			0.002	0.001	138.1469	138.1492	552.5431	552.576	1243.113	1243.255
				0.002	138.1447	138.1481	552.5126	552.5607	1242.993	1243.196
				0.003	138.1275	138.1396	552.2758	552.444	1242.069	1242.74
			0.003	0.001	138.1418	138.1467	552.466	552.538	1242.758	1243.081
				0.002	138.1396	138.1456	552.4354	552.5228	1242.638	1243.021
				0.003	138.1224	138.1371	552.1987	552.406	1241.714	1242.565
		0.190	0.001	0.001	138.1457	138.1486	552.5495	552.5791	1243.258	1243.328
				0.002	138.1393	138.1454	552.4868	552.5478	1243.139	1243.268
				0.003	138.0892	138.1207	552.0012	552.3081	1242.218	1242.813
			0.002	0.001	138.1436	138.1475	552.5179	552.5633	1243.113	1243.255
				0.002	138.1372	138.1443	552.4552	552.5321	1242.994	1243.196
				0.003	138.0871	138.1196	551.9697	552.2924	1242.073	1242.741
			0.003	0.001	138.1385	138.145	552.4407	552.5254	1242.758	1243.081
				0.002	138.1321	138.1418	552.378	552.4941	1242.639	1243.021
				0.003	138.082	138.1171	551.8927	552.2545	1241.72	1242.566

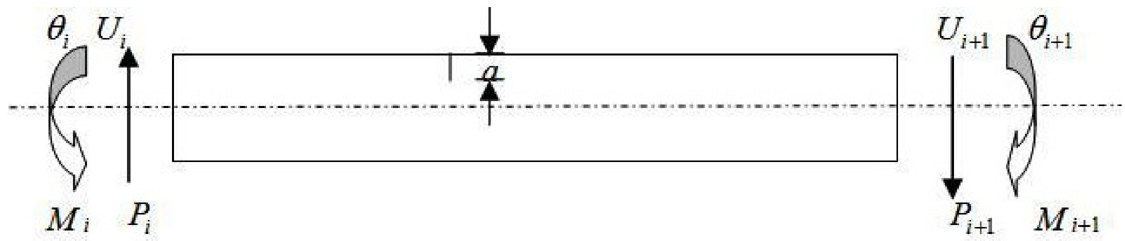

Figure 2. Schematic diagram of an element.

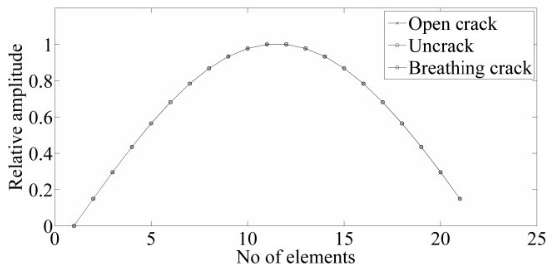

Figure 3(a). First mode of transverse vibration, s=0.5 and $l_c = 0.5$.

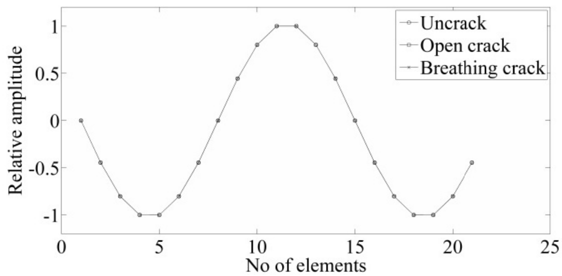

Figure 5(a).Third mode of transverse vibration, s=0.5 and $l_c = 0.05$.

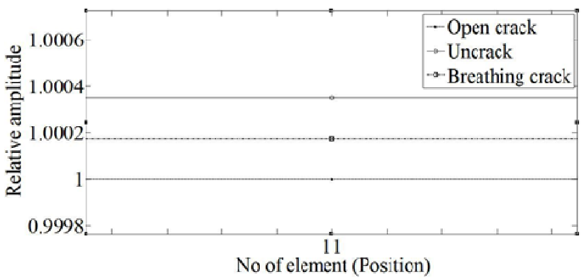

Figure 3(b). Magnified view of first mode of transverse, s=0.5 and $l_c = 0.5$.

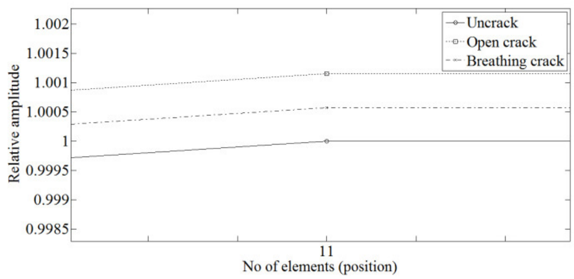

Figure 5(b). Magnified view of third mode of transverse vibration, s=0.5and $l_c = 0.05$.

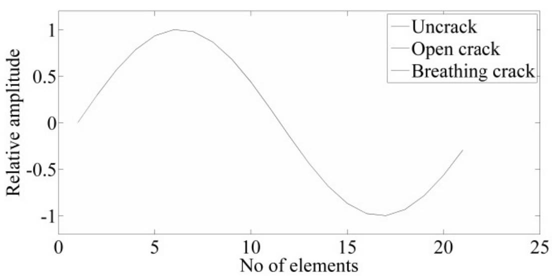

Figure 4.Second mode of transverse Vibration,s=0.5 and $l_c = 0.05$.

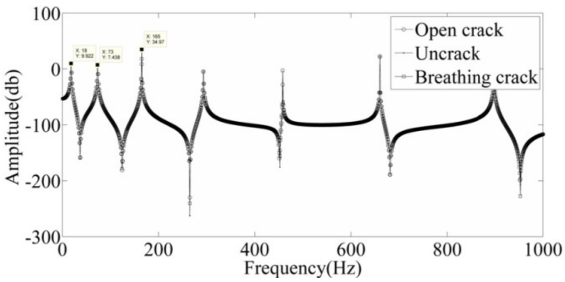

Figure 6.FRF for uncracked, open cracked and breathing cracked beam for s=0.5and $l_c = 0.05$ within the range of 0 Hz to 1000 Hz.

6. Results and Discussion

The transverse natural frequencies for the open and breathing cracked beam of mild steel are calculated and presented in Table1. It is noticed that as the relative crack depth increases the natural frequency decrease and the same for uncracked is shown in Table (2).When, the crack position shifts from the supported end to the centre, the natural frequency decreases. For deep crack (s=0.5), the difference between mode shape for breathing crack and open crack are clearly noticed, as shown in Figure 3(a) and Figure 3(b). However the effect of small crack can be obtained by magnified view (Fig. 3(b)) of the first mode shape at the crack location. Comparisons and magnified view of the transverse mode shapes of the uncracked, open cracked and breathing cracked beam are shown in Figures 3(a), 4 & 5(a) and Figure 3(b) &5(b) respectively. The Frequency response of the open cracked, uncracked and breathing cracked beam are shown in figure 6 within the frequency range of 0 Hz to 1000 Hz.

7. Conclusions

The stiffness and natural frequency decreases due to the presence of crack. The position of the crack can be identified from the deviation of mode shape between the cracked & uncracked one. For minor crack small change in mode shape takes place. The mode shape changes considerably due to the presence of crack.The FRF also changes due to the change in stiffnesss of the structure. When, the crack position shifts from the supported end to the centre, the natural frequency decreases. The most significant conclusion is that the increase in relative crack depth decreases the natural frequencies.

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